Nuclear Spin Ordering in Solid $^3$He
Pomeranchuk's Conjecture: 1950

\[ L = T \left( S_{\text{liquid}} - S_{\text{solid}} \right) \]

\[ \left( \frac{dP}{dT} \right) = \frac{S_L - S_S}{V_L - V_S} \]
Fermi-Particle Exchange Spin Interactions:

a) Large zero point energy:

\[ \psi^2 \]

\[ u = \frac{a}{3} \]

b) Atoms exchange lattice sites once in every \( \sim 10^4 \) zero point oscillations.

c) Lowest energy for two-particle exchange

- Symmetric orbital wave function
- Antisymmetric spin wave function
Early studies of exchange interactions in solid $^3$He (1960s)

**Volume dependence of $J$**

$|J| \propto V^{18}$

$J \approx -1$ mK at melting pressure

**$T_2$ in NMR**

exchange narrowing $T_2 \propto |J|$

**Magnetic susceptibility**

antiferromagnetic $\theta_W (= -4J) < 0$


R.A. Guyer, R.C. Richardson and L.I. Zane,
Rev. Mod. Phys. 43, 532 (1971)

strong volume dependence $|J| \propto V^{18}$

Thanks to Hiroshi Fukuyama
$T_{\text{initial}} = 22 \text{ mK}$
\[ \Delta P \sim \left( \frac{1}{\Delta V} \right) \int_{T_i}^{T_f} S_{\text{solid}}(T) \, dT \]
Helium Three Magnetic Phase Diagram

The diagram shows the relationship between $B$ (Tesla) and $T$ (mK) for different magnetic phases. The graph includes regions labeled HFP, LFP, and PP, indicating different magnetic phase transitions. The data points represent experimental measurements at various $B$ and $T$ values.
Multiple Exchange Hamiltonian

Even # exchanges antiferromagnetic, odd # exchanges promote ferromagnetism.

\[ J_{nn}, \quad J_t, \quad K_p \]

Frustration

$c_m = 7.7 \text{ cm/sec}$

d$P/dT = \Delta S/\Delta V$
Growth of Single Crystals of Solid 3He

a) All-liquid NMR signal
b) Heat pulse growth pattern
c) Seed XI re-growth pattern
\[
\left( \nu_i^{\pm} \right)^2 = \frac{1}{2} \left\{ \nu_L^2 + \Omega_0^2 \pm \sqrt{\nu_L^2 - \Omega_0^2}^2 + 4 \nu_L^2 \Omega_0^2 \cos^2 \Gamma_i \right\}
\]
The U2D2 Phase of Solid $^3$He

(after Star Wars' R2D2)

D.D. Osheroff, M.C. Cross and D.S. Fisher
\[
\left(\frac{\Omega_0}{2\pi}\right)^2 \times 10^{11} \text{ (Hz)}^2 \]

\[
[6.81 - 3.33 \left(\frac{T}{T_N}\right)^2] \times 10^{11} \text{ (Hz)}^2
\]

Larmor Frequency (kHz)

\[X''(2000 \text{ kHz})\]
$$\Omega_0 = \left( \frac{\sum_{i=1}^{3} v_i^4 - \nu_L^2 \sum_{i=1}^{3} v_i^2}{\sum_{i=1}^{3} v_i^2 - 2 \nu_L^2} \right)^{1/2}$$

$$\cos^2 \Gamma_i = \frac{(v_i^2 - \Omega_0^2)(v_i^2 - \nu_L^2)}{\Omega_0^2 \nu_L^2}$$

$$\Omega_0 = \left( \frac{v_i^2 (v_i^2 - \nu_L^2)}{v_i^2 - \nu_L^2 (1 - \cos^2 \Gamma_i)} \right)^{1/2}$$
Three Magnon Decay Processes

• A magnon from the upper branch (\( \omega > \gamma H \)) can decay into two magnons in the lower branch provided energy and quasi-momentum can be conserved.

• For uniform spin precession in the upper branch, this loss is manifested by an abrupt broadening of the NMR line as shown.

\[
\begin{align*}
\dot{S} &= \gamma S \times H - \lambda (d \cdot l)(d \times l) \\
\dot{d} &= d \times \left( \gamma H - \frac{\gamma^2}{\chi_\perp} S \right)
\end{align*}
\]

Thanks to Takao Mizusaki
Three Magnon Decay Processes

• Rapid three magnon decay from the upper frequency mode due to stimulated emission.

• Ohmi and Fomin showed that population of lower magnon mode produces a negative frequency shift in the upper mode.

• Non-linear spin dynamics in the U2D2 phase are complex but based on simple Hamiltonian.


Thanks to Takao Mizusaki
Domain Memory Between U2D2 and CNAF Phases

- Grow a 3-domain U2D2 sample near lower critical field.

- Measure the intensity from the three domains.

- Slowly raise magnetic field into the CNAF field Region, wait, and then lower the field back.

- Measure the intensity in the three domains again.

- For constrained xls domains are the same. For unconstrained samples, they are very different.


Thanks to Takao Mizusaki
The CNAF Phase of Solid $^3$He

Jack Hetherington
T=0 Solid $^3$He Magnetization

Henri Godfrin and David Ceperley and later Hiroshi Fukuyama
Thanks to Hiroshi Fukuyama
• Change magnetic field by a small well known amount.

• Change in melting pressure just compensates for the change in liquid and solid free energies.

• Calculate solid magnetization

• Express magnetization in terms of magnetic field scaled to change in molar volume from change in P.

• Express magnetization in terms of exchange frequencies.

D.D. Osheroff, H. Godfrin and R. Ruel

\[
\begin{align*}
H &= -(9.1 \pm 0.6)m + (23.8 \pm 2)m^3 + (7.6 \pm 2)m^5 \\
H_{C2} &= 22.3 \text{ T} \\
H &= -(6.0 \pm 1.4)m + (19.7 \pm 2)m^3 + (6.8 \pm 2)m^5 \\
H_{C2} &= 20.5 \text{ T}
\end{align*}
\]
Temperature Resolution of Solid NMR Shift

\[ \cos^2 \Gamma = 1 \]
\[ \theta = 85.2 \quad \text{crystal A} \\
80.1 \quad \text{crystal I} \\
54.3 \quad \text{crystal D2} \]
Conclusions:

BCC solid $^3$He has been a remarkably fruitful system for understanding spin ordering due to particle exchange. Two very different nuclear spin ordered phases coexist over a narrow range of magnetic field. One phase exhibits rich antiferromagnetic resonance spectra while the other allows us to rather directly determine the importance of various ring exchange processes.
Thank You for your attention!